MATH 211 Fall 2017 SECTION:

NAME: TEST I

(1) True/False Questions (2 points each): Circle T or F only. No justification is needed for this part. Here A, B etc. denote sets while $x, y, \cdots$ denote elements.

T F The power set of $\{\emptyset, \{\emptyset\}\}$ is $\{\{\emptyset, \{\emptyset\}\}, \emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

T F The set $\{\mathbb{R}, \mathbb{N}, \mathbb{Z}^+\}$ contains an infinite number of elements

T F $\{\{a\}\} \subseteq \{a, \{a\}\}$

T F The function $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(z) = z \times |z|$ is a bijection

T F if x is a real number which is not an integer, then $\lfloor x \rfloor + \lceil y \rceil = \lceil x \rceil + \lfloor y \rfloor$

T F The negation of a contingency is also a contingency

T F Let $A = \{\{a\}\}$. Then the cardinality of $A \times A$ is 1

T F The proposition $\neg p \to p \to p$ is a tautology

T F The set $\bigcap_{-\infty}^{n=0} \{n, n+1, ..., -2, -1, 0, 1, 2, \cdots, -n\}$ is finite

T F Let $f : A \to B$ be a function and T_1, T_2 be two subsets of ~~B~~ A. Then $f(T_1 \cap T_2) = f(T_1) \cap f(T_2)$

(2) (3 pt.) Show, via an element-wise proof, that the empty set is a subset of any set A. Name the method of proof you used.

(3) (4 pt.) Simplify the logical expression $\neg((p \wedge \neg q) \vee (q \wedge \neg p))$, by finding a sequence of logical equivalences that lead to the simplest possible form. You need to justify each step by naming the corresponding law of equivalence.

(4) (4 pt.) Let x be a positive real number. Show that if x is irrational then $\sqrt{x}$ is also irrational.

(5) (5 pt.) Let f be a function from a set A to a set B, and let S be a subset of A. Find an example of two sets A and B and a function f from A to B so that $f^{-1}(f(S))$ is not a subset of S. Next, find a condition that implies that $f^{-1}(f(S)) \subseteq S$. Prove your statement.

(6) (4 pt.) Given the premises $r \to \neg p$, $q \to \neg r$ and $\neg q \to p$. Show using rules of inference that one of r or $\neg r$ is a valid conclusion (only one is valid, you need to decide!). Make sure you provide the names of the rules of inference that you use.

(7) In the following use the following predicates. We define the predicates $P(x)$: "x is a professor", $C(y)$: "y is a course", $T(x, y)$: "x can teach y", $G(x)$: "x is a genius". Translate the following statements from logical expressions to English or vice versa, as appropriate. (2 points each)

(a) There is a professor who cannot teach all the courses.

(b) There are two professors who cannot teach the same course, regardless which course it is.

(c) No professor can teach all courses, unless he is a genius.

(d) $\forall x(\neg P(x) \vee \exists y(C(y) \wedge T(x, y)))$.

(e) $\exists x_1 \exists x_2(x_1 \neq x_2 \wedge P(x_1) \wedge P(x_2) \wedge \forall y(C(y) \rightarrow (\neg T(x_1, y) \leftrightarrow \neg T(x_2, y))))$